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MULTIPLE SATELLITE ORBIT DETERMINATION FOR NEIGHBORING SATELLITES

Patrick J. Fell

Naval Surface Weapons Center
Dahlgren Laboratory, Virginia

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Warfare Analysis Department

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FOREWORD

This report examines multiple satellite orbit determination for neighboring satellites. The procedures discussed can result in a reduction of both the total number of parameters and the computation time required to process satellite ephemerides individually.

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(i) The simultaneous processing of orbital data using a multiple satellite orbit determination method produces ephemerides for the satellites with an accuracy consistent with individually determined orbits. However this method predicts the relative separation of the satellites more accurately in the presence of modeling errors.

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(ii) The use of perturbation solutions from the integration of one satellite's variational equations to process the other satellite's tracking data introduces a small additional error into orbit solutions. But such a procedure can substantially reduce computation time required to compute the orbits of the satellites.

Based on this analysis it is recommended that a multiple satellite data processing scheme be adopted for satellites in neighboring orbits. In the interest of accuracy, however, the use of perturbation solution approximations should be reserved unless computation time requirements become critical.

ABSTRACT

If satellites follow orbit paths within one hundred kilometers of each other,

(i) The simultaneous processing of orbital data using a multiple satellite orbit determination method produces ephemerides for the satellites with an accuracy consistent with individually determined orbits. However this method predicts the relative separation of the satellites more accurately in the presence of modeling errors.

(ii) The use of perturbation solutions from the integration of one satellite's variational equations to process the other satellite's tracking data introduces a small additional error into orbit solutions. But such a procedure can substantially reduce computation time required to compute the orbits of the satellites.

Based on this analysis it is recommended that a multiple satellite data processing scheme be adopted for satellites in neighboring orbits. In the interest of accuracy, however, the use of perturbation solution approximations should be reserved unless computation time requirements become critical.

CONTENTS

I. INTRODUCTION

Orbit determination for a system of multiple satellites involves the determination of an extended number of parameters, usually by least squares differential correction. In some instances, depending on the orbital geometry of the system, this parameter set may be redundant. In these situations parameters that are common among various satellites may be combined to reduce the total number of state parameters required if all satellites were to be processed individually. These parameters may include dynamical quantities as well as bias parameters. A least squares method describing this parameter reduction approach is given for satellites in neighboring orbits. The method is then numerically applied to satellites which are 100 kilometers apart.

Another aspect of the orbit determination problem is the integration of variational equations to obtain perturbation solutions necessary for orbit improvement. These solutions are normally produced by a time consuming numerical integration which must be exercised for each satellite in the system. If, however, the path of two satellites are close to each other then it may be possible to utilize the perturbation solutions from one integration for both satellites. This would result in a reduction of the computation time necessary to process multiple satellites. An examination of this concept is presented below.

II. Computational Methods

A. Parameter Reduction Approach

Consider a system of multiple satellites orbiting in close proximity with a physical separation remaining less than one hundred kilometers. With this particular constellation geometry the satellites are viewed simultaneously by each tracking station. Under this condition a multiple satellite orbit determination scheme can be devised based on the assumptions that (1) the scaling constant for each satellite's atmospheric drag model are equal and (2) the refractive structure of the troposphere is equivalent for all satellites per pass. These assumptions enable the total number of state parameters to be decreased when all satellite data are processed simultaneously. This results in a single drag scaling parameter for the constellation and only one tropospheric refraction scaling parameter per pass. This reduction in the total number of estimated parameter results in a stronger covariance for the computed satellite positions.

The formulation presented in Appendix B is that designed for satellites in neighboring orbits. The technique presented may be generalized in an obvious way for other satellite systems where parameter reduction is possible.

B. Perturbation Solution Approximation

In order to form the least squares normal equations en route to determining the best ephemeris for a satellite it is necessary to produce certain quantities called perturbation solutions. These quantities as mentioned previously are produced by a time consuming numerical integration of the satellite's variational equations. In short, the variational equations express how accelerations experienced by the satellite's center of mass would change given changes in orbital constants at epoch. Numerical integration then produces a history of variations in position and velocity resulting from changes in initial orbital conditions. These partial derivatives are required for the application of the chain rule in computing the partial derivative of data taken at time t with respect to a given state parameter at epoch.

If the equations of motion for two satellites approximate each other then the differences in the perturbation solutions for the two will be small depending on how "good" the approximation is. Therefore under some conditions it is possible to utilize the perturbation solutions from one integration for both satellites:

Let t_o be some epoch time and let $\{e_j(t_o)\}_1$ and $\{e_j(t_o)\}_2$ represent the orbital elements for two satellites. Assuming that the satellites are in approximately the same orbit the idea is to determine two times t_1 and t_2 when the quantity

$$|\bar{r}_1(t_1) - \bar{r}_2(t_2)|$$

reaches a minimum during the first revolution after epoch. The vectors $\bar{r}_1$ and $\bar{r}_2$ represent the position vectors for the two satellites. Then using numerical integration update $\{e_j(t_o)\}_1$ from t_o to t_1 and $\{e_j(t_o)\}_2$ from t_o to t_2. The time t_1 and t_2 become the new epochs for the respective satellites.

Now, using a given step size Δt, a perturbed trajectory, position and partial derivatives versus time, is integrated for satellite 1 for a desired length of time. An inertial trajectory, position and velocity versus time, is integrated for satellite 2 over the same time span. A "synthetic" perturbed trajectory is then created for satellite 2 by merging its inertial trajectory with the perturbed trajectory for satellite 1. This is accomplished by extracting the time $(t_2 + n\Delta t)$ and position for satellite 2 from the inertial trajectory and the partial derivatives at time $(t_1 + n\Delta t)$ from the perturbed trajectory. This "synthetic" perturbed trajectory for satellite 2 may now be used in the least squares orbit determination process.

Obviously certain error levels will be introduced into orbit solutions due to this approximation, but the magnitude of the errors depends on the forces acting on each satellite from times t_1 and t_2 on. Numerical examples are discussed below and error estimates for this technique are given.

III. Numerical Examples

The computational procedures described above were applied to a system of three coplanar satellites in quite similiar orbits. The satellites maintain a physical separation of less than one hundred kilometers at Transit altitude (1100 kilometers). Two satellites are in the same circular orbit while the third orbit is slightly eccentric.

A. Parameter Reduction

(i) Covariance Analysis

Orbital solutions based on both one and two days of synthetic range difference data were generated to determine what effects multiple satellite data processing would have on the parameter covariance. The data noise level was twenty centimeters. For individual orbit processing using one day's data from five stations the one sigma orbit parameter uncertainty implies an orbit error of less than one meter. The one sigma uncertainty in the determination of a drag coefficient over this span of data, however, implies an orbit error of about three meters. Simultaneous processing in which a common drag parameter for all satellites is used and in which a single tropospheric refraction scaling parameter per pass is allowed yields a one sigma uncertainty in orbit parameters of some ten to twenty percent smaller than for individual processing. The one sigma drag contribution to orbit error was reduced to 1.7 meters. These results indicate that simultaneous processing is capable of reducing orbit error due to drag. This same analysis was done for two days of data. The same qualitative features resulted. It should be emphasized that this analysis is

strictly a covariance analysis. That is to say the model (geopotential, troposphere, etc.) is assumed perfect. This fact is the origin of the very small uncertainties quoted above. These results are summarized in Table 1.

(ii) Deterministic Analysis

The effects of refraction and gravity errors on both individual and simultaneous orbit determination have been examined over a two day period. Orbit solutions were based on synthetic range difference data with a standard error of twenty centimeters. Orbit solutions indicate that tropospheric refraction bias and gravity modeling (truncation) error which is recovered primarily through the drag parameter generally result in slightly smaller residuals of fit for individual processing than for simultaneous processing. Individual satellite processing incorporating both a drag parameter per satellite and a refraction bias parameter per pass is better able to "absorb" these modeling errors.

However, in the presence of these unmodeled errors simultaneous processing is better able to predict the relative separation of the satellites during a three day period following the two day fit span. Table 2 gives the error in separation distance which occurs using both processing modes for one pair of satellites. As indicated the multiple satellite processing scheme predicts the relative separation more accurately. This is graphically depicted for the case of truncation of the spherical harmonic representation of the gravity field to the 12, 12 level in Figure 1. During a three day prediction the relative

ORBIT UNCERTAINTY* FOR DIFFERENT PROCESSING METHODS

CASE	INDIVIDUAL PROCESSING	SIMULTANEOUS PROCESSING
I ONE DAY DATA SPAN:		
ORBIT PARAMETERS	< 1m	< 1 m
DRAG COEFFICIENT	3m	1.7m
II TWO DAY DATA SPAN:		
ORBIT PARAMETERS	< 1m	< 1m
DRAG COEFFICIENT	0.5 m	0.27m

* EXCLUDING EFFECTS OF UNCERTAINTIES IN THE GRAVITY FIELD.

TABLE 1

geometry was predicted more accurately when multiple satellite data processing was utilized.

B. Perturbation Solution Approximation

This method was first applied to the two satellites in a common circular orbit. A perturbation solution for the trailing satellite (satellite 2) was developed by merging its integrated inertial trajectory with the integrated perturbation solution from the leading satellite (satellite 1). This was accomplished using the procedure described above. The updated position for satellite 1 at time t_1 differed by less than ten meters from the updated position of satellite 2 at time t_2. Seven orbit fits (one iteration) were then performed using this synthetic perturbed trajectory to determine the error levels introduced by the method. These solutions were based on range difference observations over a two day period. No additional modelling errors were introduced. For each case errors in the data were introduced through an error in one initial orbit constant at a level compatible with or greater than the poorest initial condition errors in the precise Transit ephemeris determined at NSWC Dahlgren Laboratory. Table 3 gives the maximum orbit errors during both the fit span and a three day prediction period as a function of the orbit parameter perturbed. These results indicate that this method for constructing a synthetic perturbed trajectory introduces little error for the satellites in a common circular orbit. In this case the time difference between t_1 and t_2 was about twelve seconds.

RELATIVE SEPARATION ERRORS FOR 12.12 GRAVITY FIELD TRUNCATION

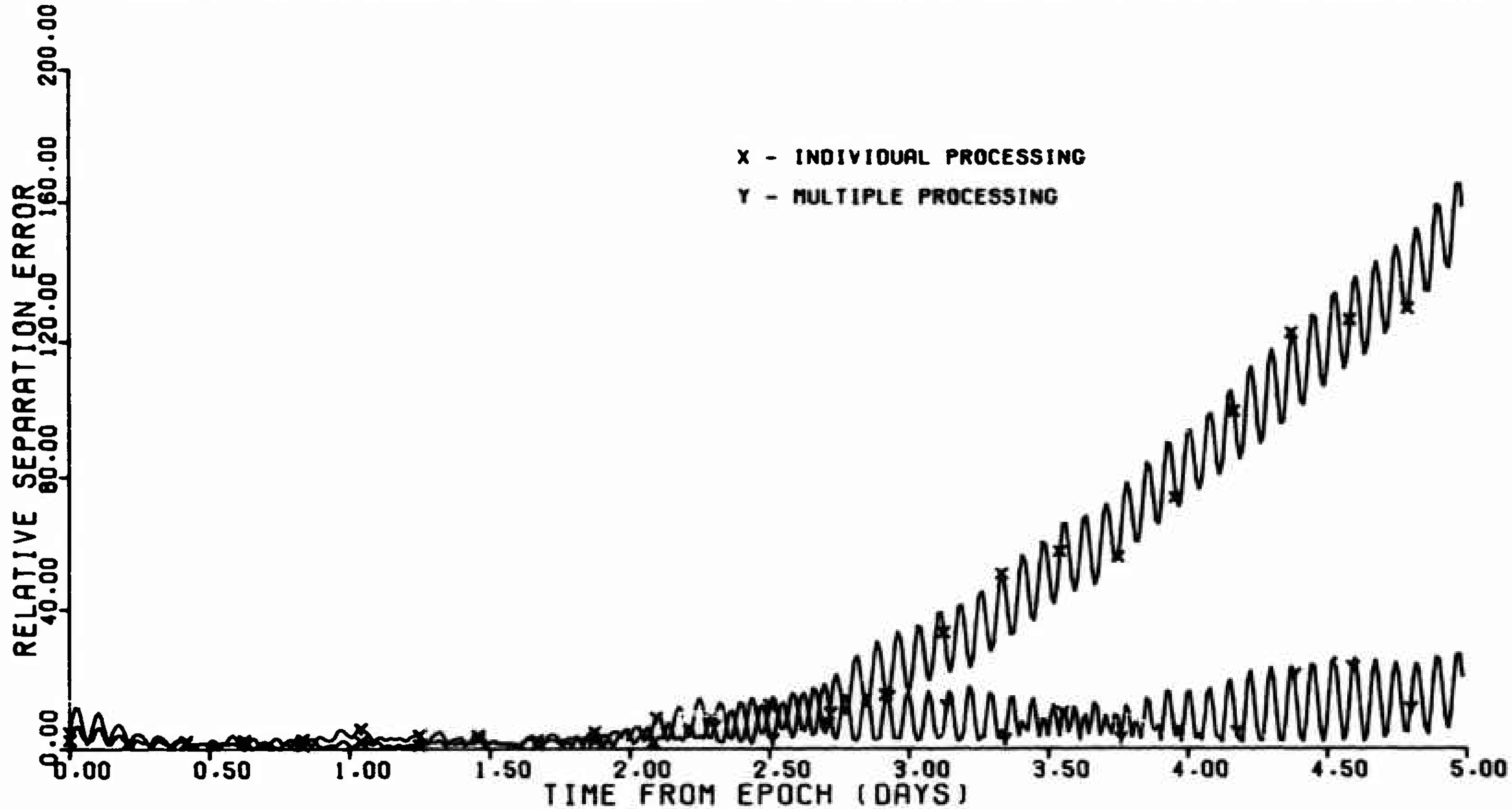

FIGURE 1

RELATIVE SEPARATION ERROR

SATELLITE PAIR	ERROR TYPE	MAXIMUM ERROR (meters)			
		INDIVIDUAL PROCESSING		SIMULTANEOUS PROCESSING	
		FIT SPAN	PREDICTION SPAN	FIT SPAN	PREDICTION SPAN
Circular & Eccentric	Gravity (12, 12 Truncation)	6	160	12	28
Circular & Eccentric	Hopfield Refraction (5% Bias)	.06	.1	.06	.1

TABLE 2

This same experiment was repeated using satellite 1 and satellite 3. The orbits in this case differ to a greater extent than the first pair considered. Using the same technique a synthetic perturbed trajectory was created for satellite 3. The minimum distance between the satellites was about 50 kilometers. In this case the time difference between t_1 and t_3 was about 60 seconds. The error levels introduced for this case are also given in Table 3.

Perturbation Solution Approximation Error Estimates

CASE	PARAMETERS PERTURBED	PERTURBATION	MAXIMUM ERROR (meters) CIRCULAR ORBIT (SATELLITE 2)		ELLIPTIC ORBIT (SATELLITE 3)	
			FIT	PREDICTION	FIT	PREDICTION
I	a	.5m	.7	4.6	1.5	6.1
II	e sin (w)	.4E−05	.3	1.2	.9	1.9
III	e cos (w)	.4E−05	.3	1.2	.7	1.1
IV	I	.5E−05 rad	.2	.9	.8	1.9
V	L+W	.8E−05 rad	.4	1.9	.9	1.4
VI	Ω	.5E−05 rad	.2	.5	.5	1.1
VII	Drag	1.0	.6	6.3	.9	8.3

TABLE 3

IV. Summary and Recommendations

This report has examined two aspects of orbit determination applicable for neighboring satellites:

(1) The simultaneous processing of orbital data using the reduced parameter set approach will produce orbits with an accuracy consistent with individual satellite orbit determination. This method however yields a better solution for the relative separation of the satellites in the presence of modeling errors.

(2) The use of perturbation solutions from the integration of one satellite's variational equations to process other satellites has been examined. Although this approach introduces additional error into orbit solutions it can substantially reduce computation time required to produce these orbits.

Based on this analysis it is recommended that a multiple satellite data processing scheme be adopted for satellites in neighboring orbits. In the interest of system accuracy however the use of any perturbation solution approximations should be limited unless computation time requirements become critical.

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APPENDIX A

Pass Matrix Concept

Pass Matrix Concept

Data Class: Range Difference (Integrated Doppler)

The range difference data class is given by

$$D(t_n) = \rho(t_n) - \rho(t_{n-1}) + f_B \frac{c}{f_s} (t_n - t_{tca})$$
$$+ \dot{f}_B \frac{c}{f_s} (t_n - t_{n-1})^2 + (1 + C_R) \left[\Delta R(t_n) - \Delta R(t_{tca}) \right] \qquad \text{(A-1a)}$$

where

$$\rho(t_n) = |\bar{r}_{sat}(t_n) - \bar{r}_s(t_n)| \qquad \text{(A-1b)}$$

and where the vectors $\bar{r}_{sat}$ and $\bar{r}_s$ are the radius vectors of the satellite and observing station respectively. The terms f_B and $\dot{f}_B$ represent the frequency bias and drift characteristic to the pass. C_R represent a refraction scaling parameter and ΔR the Hopfield tropospheric refraction correction (Reference 2). The constant f_s is the effective frequency of the satellite, c the velocity of light, and t_{tca} the time of closest approach of the satellite to the tracking station.

Data Aggregation

The pass matrix concept (Reference 3) as used in orbit determination involves aggregating satellite tracking data on a pass basis. Normal equations are formed from the tracking data of each separate pass over a station. The parameters of fit are the satellite orbital constants and certain bias parameters characteristic to the pass. These bias parameters represent three components of station position, refraction bias, range or frequency bias, and range or frequency drift.

Formation of Pass Normal Equations for Satellite "k" and Station "ℓ"

After data from the j'th pass has been edited to eliminate "bad" points, the normal equations are formed:

Let Dt_i represent the data taken at time t_i with associated standard error σ_i. The A matrix is then given by

$$A_{kj} = \begin{bmatrix} \frac{\partial D_{t_1}}{\partial P_1} & \cdots & \frac{\partial D_{t_1}}{\partial P_6} & \frac{\partial D_{t_1}}{\partial C_D} & \frac{\partial D_{t_1}}{\partial BIAS} \\ \vdots & & \vdots & \vdots & \vdots \\ \frac{\partial D_{t_n}}{\partial P_1} & \cdots & \frac{\partial D_{t_n}}{\partial P_6} & \frac{\partial D_{t_n}}{\partial C_D} & \frac{\partial D_{t_n}}{\partial BIAS} \end{bmatrix} \qquad \text{(A-3)}$$

where the parameters of fit $P_1,\ldots,P_6$, C_D, and bias terms are given in Table A-1. The weight matrix W_{kj} for the data of pass j is given by

$$W_{kj} = \begin{bmatrix} \frac{1}{\sigma_1^2} & & 0 \\ & \ddots & \\ 0 & & \frac{1}{\sigma_n^2} \end{bmatrix} \qquad \text{(A-4)}$$

With the A_{kj} and W_{kj} matrices defined the least squares normal equations for pass j and given by:

$$B_{kj}\,\overline{\Delta P}_{k\ell} = \overline{E}_{kj} \qquad \text{(A-5)}$$

$$B_{kj} = A_{kj}^T\, W_{kj}\, A_{kj} \qquad \text{(A-6)}$$

$$\overline{E}_{kj} = A_{kj}^T\, W_{kj}\, \overline{\delta}_{kj} \qquad \text{(A-7)}$$

PARAMETERS OF FIT

NOTATION	PARAMETER
Dynamical Terms:	
P_1	x or a
P_2	y or $e \sin \omega$
P_3	z or $e \cos \omega$
P_4	$\dot{x}$ or I
P_5	$\dot{y}$ or $L+G$
P_6	$\dot{z}$ or Ω
C_D	Drag
Bias Terms:	
x, y, z	Station Position
C_R	Refraction Bias
R_B or f_B	Range or Frequency Bias
$\dot{R}_B$ or $\dot{f}_B$	Range or Frequency Drift

TABLE A-1

where the nxl vector $\bar{\delta}_{kj}$ contains the observational residuals. The pair $[B_{kj}, E_{kj}]_\ell$ are denoted as the pass matrix for the j'th pass.

APPENDIX B

Multiple Satellite Least Squares

Multiple Satellite Least Squares

Least Squares Method

Let $[B_{kj}, \bar{E}_{kj}]_\ell$ denote the pass matrix (Appendix A) formed from the edited observations from the j'th pass of satellite k(k=1,2,3) over station ℓ. These normal equations

$$B_{kj} \; \overline{\Delta P}_{k\ell} = \bar{E}_{kj} \quad k=1,2,3 \tag{B-1}$$

are formed for the orbital and bias parameters associated with the satellite-station pair $k\ell$ (Table A-1). A typical pass matrix will be of the form:

$$[B_{kj}, \bar{E}_{kj}]_\ell \equiv [B,\bar{E}]_{kj\ell}$$

$$= \begin{bmatrix} B_{o,o} & B_{o,C_D} & B_{o,b} & \bar{E}_o \\ & B_{C_D,C_D} & B_{C_D,b} \; , & \bar{E}_{C_D} \\ * & & B_{b,b} & \bar{E}_b \end{bmatrix}_{kj\ell} \tag{B-2}$$

where the subscript "o" refers to the orbital parameters, C_D refers to the drag scaling parameters, and the subscript "b" refers to pass dependent bias parameters.

*symmetric

For each pass j the elements of the pass matrix $[B,\overline{E}]_{kj\ell}$ (k=1,2,3) are permuted so that certain pass dependent parameters (station and frequency bias) may be formally eliminated. The equations for the elimination of these pass dependent parameters are

$$B_\alpha \overline{\Delta P}_\xi + B_\beta \overline{\Delta P}_\eta = \overline{E}_\xi \tag{B-3a}$$

$$B_\gamma \overline{\Delta P}_\xi + B_\delta \overline{\Delta P}_\eta = \overline{E}_\eta \tag{B-3b}$$

where the matrices $B_\alpha,\ldots,B_\delta$ and $\overline{E}_\xi$ and $\overline{E}_\eta$ are constructed by subdivision of the matrices of equation (B-2):

$$B_\alpha = \left[\begin{array}{c:c:c} B_{o,o} & B_{o,C_D} & B_{o,C_R} \\ \hdashline & B_{C_D,C_D} & B_{C_D,C_R} \\ \hdashline * & & B_{C_R,C_R} \end{array}\right]_{kj\ell} \quad (8\text{x}8) \tag{B-4a}$$

$$B_\beta = \left[\begin{array}{c:c} B_{o,xyz} & B_{o,f_B} \\ \hdashline B_{C_R,xyz} & B_{C_R,f_B} \end{array}\right]_{kj\ell} \quad (8\text{x}4) \tag{B-4b}$$

$$B_\gamma = B_\beta^T \tag{B-4c}$$

$$B_\delta = \left[\begin{array}{c:c} B_{xyz,xyz} & B_{xyz,f_B} \\ \hdashline B_{f_B,xyz} & B_{f_B,f_B} \end{array}\right]_{kj\ell} \quad (4\text{x}4) \tag{B-4d}$$

*symmetric

$$\overline{E}_{\xi} = \begin{bmatrix} \overline{E}_{o} \\ \hdashline \overline{E}_{C_D} \\ \hdashline \overline{E}_{C_R} \end{bmatrix}_{kj\ell} \quad (8x1) \tag{B-4e}$$

$$\overline{E}_{\eta} = \begin{bmatrix} \overline{E}_{xyz} \\ \hdashline E_{f_B} \end{bmatrix}_{kj\ell} \quad (4x1). \tag{B-4f}$$

The parameters contained in the state vector $\overline{\Delta P}_{\eta}$ are those being eliminated. The vector $\overline{\Delta P}_{\xi}$ contains all other parameter corrections. Solving equation (B-3b) for $\overline{\Delta P}_{\eta}$ and substituting into equation (B-3a) one obtains the eliminated form of the normal equations:

$$(B_{\alpha} - B_{\beta} B_{\delta}^{-1} B_{\gamma}) \overline{\Delta P}_{\xi} = \overline{E}_{\alpha} - B_{\beta} B_{\delta}^{-1} \overline{E}_{\eta} \tag{B-5a}$$

or

$$B'_{kj} \, \overline{\Delta P}'_{k\ell} = \overline{E}'_{kj} \qquad (k=1,2,3) \tag{B-5b}$$

$$\overline{\Delta P}'_{kj} \equiv \overline{\Delta P}_{\xi} \tag{B-5c}$$

where the prime indicates the eliminated normal equations. For each pair kj (k=1,2,3) we now have the eliminated pass matrices $[B',\overline{E}']_{kj\ell}$ for the parameter set $\overline{\Delta P}'_{k\ell}$.

Now, under the assumption that the atmospheric drag scaling constants and tropospheric refraction correction parameters per pass are common among all three satellites, the normal equations for the j'th pass may be

combined to yield the combined pass matrix $[B', \bar{E}']_{j\ell}$:

$$[B',\bar{E}']_{j\ell} = \tag{B-6}$$

$$\left[\begin{array}{ccc|c|c|c}
B'^{2}_{o,o} & 0 & 0 & B'^{1}_{o,C_D} & B'^{1}_{o,C_R} & E'^{1}_{o} \\
 & B'^{2}_{o,o} & 0 & B'^{2}_{o,C_D} & B'^{2}_{o,C_R} & \bar{E}'^{2}_{o} \\
 & & B'^{3}_{o,o} & B'^{3}_{o,C_D} & B'^{3}_{o,C_R} & \bar{E}'^{3}_{o} \\
 & & & \sum\limits_k B'^{k}_{C_D,C_D} & \sum\limits_k B'^{k}_{C_D,C_R} & \sum\limits_k \bar{E}'^{k}_{C_D} \\
 & * & & & \sum\limits_k B'^{k}_{C_R,C_R} & \sum\limits_k \bar{E}'^{k}_{C_R}
\end{array}\right]_{\ell}$$

At this point the elimination equations may again be utilized to eliminate the pass dependent tropospheric refraction scaling parameter C_R yielding a new set of eliminated pass matrices involving only dynamic parameters:

$$B''_{j(19x19)} \; \overline{\Delta P''}_{(19x1)} = \bar{E}''_{j(19x1)} \; . \tag{B-7}$$

At this point all pass normal equations (B-7) may be summed to yield the final arc normal equations for the solution of dynamical parameters (six per satellite along with a common drag for the three satellites):

$$\left(\sum_{j=1}^{N} B''_j \right) \overline{\Delta P''} = \sum_{j=1}^{N} \bar{E}''_j \tag{B-8}$$

*symmetric

or

$$\overline{\Delta P}'' \left(\sum_{j=1}^{N} B''_j \right)^{-1} \left(\sum_{j=1}^{N} E''_j \right) \tag{B-9}$$